Conference

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ABTRACTS

Coordinate free expressions of exponentials of multivectors in $Cl(p, q)$ for $p + q = 3$

Acus Arturas

Lithuania

Closed form coordinate free expressions to calculate the exponential of a general multivector (MV) in Clifford geometric algebras (GAs) $Clpq$ are presented for $n = p + q = 3$. The obtained exponential formulas were applied to find exact GA trigonometric and hyperbolic functions of MV argument.

A Conformal Geometric Algebra approach for sphere and spherical shell intersections in $\mathbb{R}^n$

Alves Rafael

Brazil

Sphere intersections are involved in several real problems, such as molecular modeling, global positioning systems, data science among others. When the data provided has uncertainties, the radii of the spheres involved are not precise, which motivates the use of spherical shells. With the help of GAALOP, we provide a CGA approach to intersect spheres and spherical shells in any dimension and compare with linear algebra approaches.

Dirac Equation Redux by Direct Quantization of the 4-Momentum Vector

Andoni Sokol

Denmark

Dirac equation (DE) is a cornerstone of quantum physics. We prove that direct quantization of the 4-momentum vector p of modulus m (the rest mass) yields a coordinate-free and manifestly covariant equation, which in coordinate representation is equivalent to DE with spacetime frame vectors replacing Dirac's gamma-matrices. We augment spacetime to spacetime-reflection (STR) and develop the formalism by deriving all the essential results from the novel STR DE: conserved probability currents, symmetries, nonrelativistic approximation and spin 1/2 magnetic angular momentum. It will become clear that key symmetries follow more directly and with clearer geometric interpretation in STR than in the standard approach. In simple terms, we demonstrate how Dirac matrices are a redundant representation of spacetime-reflection directors.

Spin-1/2 one- and two- particle systems in physical space without eigen-algebra or tensor product

Andoni Sokol

Denmark

A novel representation of spin 1/2 combines in a single geometric object the roles of the standard Pauli spin vector operator and spin state. Under the spin-position decoupling approximation it consists of three orthonormal vectors comprising a gauge phase. In the one-particle case the representation: (1) is Hermitian; (2) shows handedness; (3) reproduces all standard expectation values, including the total one-particle spin modulus, etc. In the two-particle case: (1) entangled pairs have precisely related gauge phases; (2) the dimensionality of the spin space doubles due to variation of handedness; (3) the four maximally entangled states are naturally defined by the four improper rotations in 3D: reflections onto the three orthogonal frame planes (triplets) and inversion (singlet). All standard expectation values for 'on paper' (controlled gauge phase) and 'on lab' (uncontrolled gauge phase) spin transformations are derived. Spin is directly defined and transformed in 3D orientation space, without use of eigen algebra and tensor product as in the standard formulation.

Geometric Algebra for Operation with Physical Directions

Andresen Jens

Denmark

An Epistemology for Geometric Algebra Obeying the Structures of Nature: The idea is, looking out at a star in the celestial sky, you immediately perceive a direction. This we can measure in two plan angles (Azimuth, Altitude) relative to the Earth. The Earth is rotating, hence the direction to the star is rotating. We represent such directions with a line like generator 1-vectors, and we use them as multiplication operators in a Geometric Algebra. We presume the direction concept to be universal, that mean that direction to a distant galaxy is locally inherit down to the tiniest subatomic elementary particle. Elementary particles can be treated as rotation multiplications of directions, just as astronomic directions.By synthetic judgment we presume the foundation for all directions are absolute engraved in a relative universe, that is translation and boost invariant. The relative relation between two directions is a conform plan angle absolute measure part of the hole plane circle circumference 2. There is a minor problem, all information about the surroundings of each location is hesitating and retarded by the fundamental isotropic speed of information, which also applies to light ray directions. This we dissolve through the null vector idea of light-like-curve-lines in some Minkowski-space substance with both; three positive signature bivector dimensions, and three negative signature bivector dimensions in a Non-Euclidian Space-Time-Algebra as David Hestenes introduced 1966. The orthonormal basis of three positive signature subject bivectors is projected by multiplication through the information development dimension (time) to an ordinary objective Euclidian space with three extensive Cartesian dimensions, we imagine as objects along the edge of a cube. This traditional three dimensional space on Earth was already describe by Aristoteles (384-22 BC) as: Length, Breadth, Depth. The fundamental categorical idea in physics is the concept direction.

Time Harmonic Relativistic Wave Mechanics

Arsenovic Alexander

USA

This paper presents an approach to special relativity which is more in line with electrical engineering technique, namely as the time-harmonic analysis of a linear system. The approach is derived from Hestenes' Zitter model for the electron, which assumes an internal structure of a point-particle following a light-like helix in spacetime. By making a small adjustment to the concept of a differential and definition of velocity, a time-harmonic model is constructed which produces multivector equations equivalent to the conventional scalar versions. In addition, the grade (dimension) of the quantities involved match their units. The vacuum constants μ_0 and ϵ_0 are found to be a consequence of electrodynamics.

Innovative Conformal Geometric Implications for Computer Aided Design

Bell Ian

UK

Conformal Geometry, intuitively defined by generalizing planar reflections to spherical inversions, and exploiting spherical as well as vector arithmetic, provides revolutionary rotations around circular and compound axies and meaningful interpolation of reflections with profound implications for Computer Aided Design, much of which is still effectively confined to rigid 2.5D. There are difficulties, such as the ability to turn BREPS inside out and the creation of less manufacturable designs, but the generalisation of mates from rigid to conformal, and the generalization of parameterized surfaces to parameterized medial solids in particular, are potential game changers.

A Representation of the Geometric Product for 3-Vectors by Means of Matrix Algebra

Bongardt Bertold

Germany

We outline how a specific product with a symetric matrix representation is complemented by a suitable antisymmetric product such that the sum of these matrix products yields a real-number matrix representation of the geometric product for the case of vectors in three-dimensional space. To the best of the authors' knowledge, this direct correspondence of 'abstract' geometric algebra with 'concrete' matrix algebra has not been documented yet. At first the basic observation may be useful for practical computations, at least it opens a door for further research. Finally, the connection underlines the relevance of geometric calculus as a powerful tool for various fields of research and engineering.

First insights into Autoencoders in Geometric Algebra

Buchholz Sven

Germany

Many Neural Networks (deep or not) have been successfully formulated in the Complex and Quaternionic domain, indeed generally in Geometric Algebra (GA). In recent years autoencoders, i.e.unsupervised learning regimes, have also gained attention in that sense, see e.g. [1], [2] for the Complex domain, [3] for Quaternions. We study how, based on these works, autoencoders can be formulated in the general realm of GA, and, what could be the prospects of such an approach.

[1] Baldi and Lu: Complex-Valued Autoencoders, `https://arxiv.org/abs/1108.4135`.

[2] Azizi, Wandel, and Behnke: Complex Valued Gated Auto-encoder for Video Frame Prediction, `https://arxiv.org/abs/1903.03336`.

[3] Grassucci, Comminiello, and Uncini: A Quaternion-Valued Variational Autoencoder, `https://arxiv.org/abs/2010.11647`.

Implementation of the Tube Elbow Detection Based GAC Algorithm

Byrtus Roman

Czechia

We provide an implementation of a tube elbow detection algorithm based on the Geometric Algebra for Conics (GAC) applied to data obtained from a simulation running in a game engine. Using the algorithm we estimate parameters of an elbow, i. e. the curvature and the direction along the centerline of the tube. As a result, we reconstruct the tube's central axis line which can be used for a robot's autonomous navigation in the form of a path following algorithm.

The adjoint problem in astrodynamics. Clifford algebra and Riemannian geometry

de Niem Detlef

Germany

The controlled Kepler problem is the basic problem of astrodynamics. For low thrust e.g. cargo flights from the Earth to the Moon multiple revolutions are required (remember ESA's SMART 1 to the Moon). I will show that in the adjoint problem of Pontrygin's maximum principle there appears a Clifford algebra. Another method is to use some kind of Elements as variables. This allows to eliminate the adjoint variables and to replace the entire problem with a geodesic equation with source term in a 6-dimensional Riemannian space. If time restrictions allow I will also indicate what to do in the planar CRTB (circular restricted problem of 3 bodies) case.

Solver-free optimal control for Linear Dynamical Switched System by means of Geometric Algebra

Derevianko Anna

Czechia

We design an algorithm for control of a linear switched system by means of Geometric Algebra. More precisely, we develop a switching path searching algorithm for a two-dimensional linear dynamical switched system with non-singular matrix whose integral curves are formed by two sets of centralised ellipses. Then it is natural to represent them as elements of Geometric Algebra for Conics (GAC) and construct the switching path by calculating the switching points, i.e. intersections and contact points. For this, we use symbolic algebra operations, more precisely the wedge and inner products, that are realisable by sums of products in the coordinate form. Therefore, no numerical solver to the system of equations is needed. Indeed, the only operation that may bring in an inaccuracy is a vector normalisation, i.e. square root calculation. The resulting switching path is formed by pieces of ellipses that are chosen respectively from the two sets of integral curves. The switching points are either intersections in the first or final step of our algorithm, or contact points. This choice guarantees optimality of the switching path with respect to the number of switches. On two examples we demonstrate the search for conics' intersections and, consequently, we describe a construction of a switching path in both cases.

The structure of the Kerr singularity

Doran Chris

UK

In previous work it was shown that the Kerr singularity has a disk structure with two properties: it is rigidly rotating and has the stress-energy tensor of an ideal isotropic body. In this talk we reverse the logic and derive the full Kerr solution starting from this source singularity. This is the first time that the Kerr solution has been derived entirely from the properties of its source.

Repeated Quantum Prisoner's Dilemma based on subgroups of Spin(6)

Eryganov Ivan

Czechia

The quantum version of the repeated Prisoner's Dilemma employing complex Clifford algebras framework will be formulated in the article. The repeated version consisting of two game iterations will be modeled with the help of the Spin(6) group. Analysis of the Nash Equilibrium will be performed with respect to different values of entanglement parameters and players' possible strategy spaces, which will correspond to the particular subgroups of the Spin(6).

Conformal Geometric Algebra applied to the Discretizable Molecular Distance Geometry Problem with arbitrary dimension

Fidalgo Felipe

Brazil

The Distance Geometry Problem (DGP) is an inverse problem that seeks conformations of a simple and undirected graph G whose edges are weighted by a real distance function. This lecture deals with the K-Discretizable Molecular Distance Geometry Problem (K-DMDGP), a subclass of the DGP in the Euclidean $\mathbb{R}^K$ for whom the search space is discrete and can be designed as a binary tree, and presents a description of such problem completely in terms of the Conformal Geometric Algebra (CGA) and, also, a theoretical approach for a method to handle this in this fashion. It is a joint work with Valter S. Camargo, Emerson V. Castelani and Leandro A. F. Fernandes.

Switching Extended Kalman Filters for Rotation Estimation

Grafton Alex

UK

In this paper, we explore the performance of the switching extended Kalman filter, focussing on process and measurement function selection for rotation estimation. The switching extended Kalman filter is able to switch between process functions and measurement functions. Process functions include constant velocity or acceleration in rotation parameter space, or constant body-frame angular velocity. Measurement functions specify how rotation parameters are mapped to a rotor. Changing this mapping alters the meaning of the rotation parameters in the state and the behaviour of process functions that operate in rotation parameter space. We explore how the changes in this meaning can be appropriately handled and demonstrate how the switching extended Kalman filter offers improved robustness to misconfiguration.

Emergent Spin of Multivector Fields

Greenwood Ross

USA

Is a quantum field's spin wholly determined by the form of the kinetic term in its Lagrangian? Analogous to the Higgs mechanism "granting" mass to otherwise massless fields, this work motivates and explores a construction of multivector fields in which spin and helicity may be endowed by the presence of interaction terms. We draw a comparison between this scheme and elements of the Weinberg-Salam electroweak model.

Adapting Matrix Algorithms for Multivectors

Hadfield Hugo

UK

In this talk we will look at the problem of computing certain functions of multivectors for which algorithms already exist for matrices but which do not have a basis free formulation applicable to direct use in geometric algebra software. Taking the examples of the square root function and the logarithm we consider commonly used matrix algorithms, how to modify them to work on multivectors in a coordinate free way, and describe the challenges that remain in bringing the tools of computational matrix algebra to Clifford algebra software libraries.

Computing with Algebras over Algebras

Hadfield Hugo

UK

When working with the Clifford Algebras it is common to have coefficients that are in the real numbers, complex numbers, quaternions and in certain recent applications, dual numbers. Here we look at the practical applications of algebras with coefficients drawn from various different algebras and arithmetic systems, including the clifford algebras themselves. We look at the pros and cons of computing with these algebras including applications involving high precision arithmetic and higher order forward mode automatic differentiation.

Bell's Theorem – a Geometric Perspective

Held Carsten

Germany

The Bell-CHSH inequality can be used to show that no local hidden-variable theory can reproduce the correlations predicted by quantum mechanics (QM). It can be proved that certain QM correlations lead to a violation of the classical bound established by the inequality. This is Bell's Theorem. Here, we show that the theorem depends crucially on the assumption that the values of physical magnitudes are scalars. More specifically, the assumption that these values are not scalars, but vectors that are elements of the geometric algebra G3 over R3, makes it possible that the classical bound is violated even given a locality assumption.

Error-Free Geometric Algebra

Honorio Araujo da Silva David William

USA

We propose a novel approach to general-purpose error-free computations with Geometric Algebra (GA). Our main contribution is based on the consideration of Clifford signatures over a special set of integers known as Hensel codes. This particular set of integers can be described as a finite field isomorphic to a specialized set of irreducible fractions defined in terms of an odd prime number. The cardinality of that set of fractions equals the chosen prime number, and therefore, any sufficiently large odd prime will define a set of irreducible fractions that includes the inputs as well as the solutions for any problem that GA can handle. The main benefit of this approach is the ability to perform GA computations over integers representing a subset of rational numbers without involving rounding errors. This is particularly instrumental for classes of ill-conditioned problems and numerically unstable algorithms that cannot tolerate rounding errors during the computation of any given solution. As a secondary contribution, we propose an isomorphic mapping between Hensel codes and multivectors of the two-dimensional geometric product space, allowing one to replace rational arithmetic (over a subset of rational numbers) with the two-dimensional GA over the integers.

Geometric algebras in mathematics control theory

Hrdina Jaroslav

Czechia

Currently, geometric algebras are not sufficiently established in control theory although they naturally appear. However, certain control systems are based on Grassmann algebras $\mathbb{R}^n \oplus \wedge^2\mathbb{R}^n$ and their subalgebras together with a Euclidean scalar product on $\mathbb{R}^n$. Indeed, by using GA we can exploit the properties of the simply connected group $Spin(n)$ to design particular control algorithms in a very efficient way.

Quantum Game Theory using Geometric algebra

Chappell James

Australia

N-player quantum games are analysed using Geometric algebra. We avoid the standard quantum mechanical approach involving unitary transformations on state vectors in a Hilbert space, through using rotors and multivectors in geometric algebra. Employing an Einstein-Podolsky-Rosen (EPR) setup, we find that the N-player case becomes tractable. The new mathematical approach presented here has implications in the areas of quantum information and quantum complexity, as it opens up a new way to tractably analyse N-partite qubit interactions. A comment is also made on the implied expanded view of spacetime using geometric algebra.

Geodesics in eight-dimensional spacetime

Chappell James

Australia

Minkowski Spacetime can be represented by a subspace of the Clifford algebra of three dimensions $C\ell(\Re^3)$, an eight dimensional graded algebra. This representation allows a generalization of the invariant Minkowski spacetime distance as well as the Lorentz transformations. It also has the important property of unifying the conventional four dimensions of space and time with spin and helicity. In this paper, we investigate the geodesics in this expanded spacetime and produce a generalized Lorentz force law, which can model a range of known forces including electromagnetic and general relativistic motion.

Quantum correlations are weaved by the spinors of the Euclidean primitives

Christian Joy

United Kingdom

I will review my approach to local-realistically underpinning the origins and strengths of quantum correlations using geometric algebra. My approach turns out to be related to the "1d up approach" to conformal geometric algebra advocated by Prof. Anthony Lasenby in a different context. The framework I have proposed is based on an interplay between the quaternionic 3-sphere and an octonion-like 7-sphere. It circumvents Bell's theorem by allowing a locally causal underpinning of quantum correlations, without requiring backward causation, super-determinism, or any other conspiracy loophole. After reviewing my approach, I respond to its partial critique by Prof. Lasenby.

Algorithms for multi-conditioned conic fitting in Geometric algebra for conics

Loučka Pavel

Czechia

We introduce implementations of several conic fitting algorithms in Geometric algebra for conics. Particularly, we incorporate additional conditions into the optimisation problem, such as centre point position at the origin of coordinate system, axial alignment with coordinate axes, or, eventually, combination of both. We provide mathematical formulation together with the implementation in MATLAB. Finally, we present examples on a sample dataset and offer possible use of the algorithms.

Computing medical robotics algorithms in the conformal geometric algebra

Martinez-Terán Gerardo

Mexico

Design of algorithms in the conformal geometric algebra for kidney minimal-invasive surgery tasks of haptic based robot medical systems. These includes trajectory panning, kinematic modelling and a robust-adaptable control without involving the plant dynamics which allows the global control for local kinesthetic-guided tasks under uncertainties and perturbations.

Standard Model Gauge Theory Applied to a Dirac-like Equation in $G_{4,1}$

McClellan Gene

USA

Recent work resulted in a Dirac-like equation of a specific form in $G_{4,1}$ having distinguishable field solutions corresponding to chiral states of free, plane-wave electrons, neutrinos and their antiparticles. Furthermore, the investigation found raising and lowering operators in this geometric algebra providing a Fock-space ladder relationship among the states of this single equation appropriate for the electroweak sector of the Standard Model (SM) of particle physics. The ladder of states respects the SU(2) chiral asymmetry of the SM. This presentation will explore the application of gauge theory to this specific Dirac-like equation.

Handedness, Chirality and Orientation in Relativistic Quantum Mechanics

McClellan Gene

USA

Relativistic quantum mechanics and the Standard Model of elementary particle physics add significant features to the Pauli spin states traditionally used to illustrate quantum measurements and entanglement. Geometric algebra helps illuminate this quantum landscape. Working in geometric algebra and calculus, this presentation will review the concepts introduced in transitioning from the Pauli equation to the Dirac equation and then to a Dirac-like equation incorporating both electron and neutrino states. The presentation will address the SM chiral description of the electron and neutrino fields and the behavior of chirality during spacetime propagation of the electron field. It will discuss the relationship among handedness, chirality and orientation in the context of the electron field and the space in which the field is produced and propagates.

Comparing Trajectories for Biomedical Analysis

Moroz Matt

UK

In analysing human motion for medical diagnosis and treatment we often need to compare and assess the similarity between multiple trajectories in 3D space. There exist a variety of metrics that can be used to compare these paths through space, eg. Frechet distance and dynamic time warping methods. The selection of which method to use is not always immediately obvious. In this talk we consider the properties of several of these metrics and propose novel methods based on geometric algebra. We look at the particular example of Epley's manouvre for Benign Paroxismal Positional Vertigo (BPPV).

Quantum computing based on complex Clifford algebras

Návrat Aleš

Czechia

The use of complex rather than real Clifford algebra seems natural since the quantum mechanics is intrinsically complex. We show how to represent both n-qubits and quantum gates acting on them as elements in the same complex Clifford algebra defined on a complex vector space of dimenision 2n.

Wedge product and Geometric Quantification of Entanglement

Panigrahi Prasanta

India

Entanglement is one the fundamental quantum correlations, having a wide range of applications in performing quantum information tasks. Recently, the quantification of this correlation using geometric methods [1,2] has gained tremendous attention. The formulation of I-Concurrence in terms of wedge product formalism [1] using Lagrange-Brahmagupta identity provides a deep insight to visualise entanglement in terms of area of the parallelogram described by the post measurement vectors. The separability of a pure quantum state corresponds to parallelism of the post measurement vectors and the orthogonality and equality of the post-measurement vectors maximize the entanglement [3]. This approach also provides a proper extension of the monogamy of tripartite entanglement in terms of inequality of area of parallelograms in a two dimensional subspace [4].

Learning Rotations

Pepe Alberto

UK

Several problems in computer vision are solved via deep learning. Tasks such as pose estimation from images, pose estimation from point clouds or structure from motion all imply a regression on rotations. However, 3 and 4D representations of 3D rotations, including Euler angles, axis-angle and quaternion representations, have been shown to produce significant errors even after successful training of the network. This limitation is interpreted by Zhou et al. as a topological issue related to the continuity of the rotation representation. We question this argument of continuity of the rotation representation, instead suggesting a full geometric algebra description of rotations for regression tasks via neural networks. We reproduce and validate previous results through an autoencoder sanity test and a rotation estimation between 3D point clouds. We showe that a 3D bivector representation of 3D rotations is comparable to a previous 6D continuous representation in terms of regression error and reconstruction accuracy. The high geodesic error previously explained as a discontinuity in the representation is absent when using a full rotor description of rotations and a loss function that does not imply the conversion of the chosen representation to the 3 x 3 rotation matrix. In addition, the bivector representation has been shown to be more reliable than the 6D representation when estimating rotations between noisy point clouds, both in terms of geodesic error and in distance between predicted and test point clouds. Geometric algebra hence provides a framework for describing rotations that is suitable for regression tasks via deep learning with high estimation accuracy and high generalizability in realistic high-noise scenari.

A Framework for a Multidimensional Analytic Signal

Sangston Kevin James

USA

This work examines the problem of extending the one-dimensional analytic signal, which is ubiquitous throughout signal processing, to higher dimensional signals. Bulow et al. and Felsberg et al. use techniques from Clifford algebra and analysis to extend the one-dimensional analytic signal to higher dimensions. However, each author sets forth a different definition of a multidimensional analytic signal. Herein we follow Brackx et al. and adopt a general definition of an analytic signal to show how both the hypercomplex signal of Bulow et al. and the monogenic signal of Felsberg et al. may be obtained and extended within the same mathematical framework. The crux of our approach is captured by the following statement: A generalized analytic signal is generated by an idempotent. In the manuscript/presentation we will develop this notion more specifically using examples from Clifford algebra.

The 4D nearest rotation matrix problem

Sarabandi Soheil

Spain

In this paper, we address the problem of restoring the orthogonality of a numerically noisy 4D rotation matrix by finding its nearest (in Frobenius norm) correct rotation matrix. This problem can be straightforwardly solved using the Singular Value Decomposition (SVD). Nevertheless, to avoid numerical methods, we present two new closed-form methods. One relies on the direct minimization of the cost function in terms of 4×4 matrices, and the other on the passage to double quaternion representation. A comparison of these two methods with respect to the SVD reveals their superiority in all aspects. Remarkably, their computational costs are almost two orders of magnitude faster than the SVD.

On Lie groups defining inner automorphisms that leave invariant fundamental subspaces of geometric algebra

Shirokov Dmitry

Russia

We consider inner automorphisms that leave invariant fundamental subspaces of real and complex Clifford (geometric) algebras' subspaces of fixed grades and subspaces determined by the reversion and the grade involution. We present groups of elements that define such inner automorphisms and study their properties. Some of these Lie groups can be interpreted as generalizations of Clifford, Lipschitz, and spin groups. We study the corresponding Lie algebras. The talk is based on the paper `https://doi.org/10.1007/s00006-021-01135-6` and some recent results.

The Geometric Algebra Lift of Qubits and Beyond

Soiguine Alexander

USA

The suggested formalism demonstrates that the core of quantum computing should not be in entanglement, as common wisdom reads, which only formally follows from representation of many particle states as tensor products of individual states. The core of quantum computing schemes should be in in dealing with quantum states as operators acting on observables, all formulated in geometric algebra terms. In that way quantum computer will be a kind of analog computer processing information through objects with infinite number of degrees of freedom, contrary to the two-dimensional Hilbert space elements.

Robotic manipulator based on conformal geometric algebra

Stodola Marek

Czechia

The paper deals with Conformal geometric algebra, which is applied to the calculation of forward kinematics of a robotic manipulator UR10 from Universal Robots. It is also applied to determine the position of the machine based on the location and rotation of two cameras. Then it is used in an inverse task, where based on records from the two cameras, dimensions of the UR10 manipulator and possibilities of its movement, the mutual position of these cameras is determined. And consequently the possibilities of their location in space.

A Comparative Study For Linear Circuit Analysis By Geometric Algebra And Harmonic Domain

Sundriyal Nitin

Mexico

Analysis of electric circuit in non-sinusoidal, non-linear condition has been a talking point for a long time now. Many scientific communities hold different ideologies about the additional analysis tool and domain which has resulted in different standards and definitions. Since beginning up till the date, the electric power system is getting complex with the addition of Power Electronic equipment, converters and Renewable Energy sources. All these power electronic equipment has revolutionalised the power system and brought numerous benefits to the industrial application. Still, unfortunately, this is at the cost of distortion (voltage and current) in our power system. This necessitates the understanding of power flow in non-sinusoidal linear & non-linear circuit condition. There is always a need of a novel mathematical framework to analyze the circuit in such an environment so that a consensus can be achieved regarding the standards that can comply with the well known established standards in sinusoidal condition. The work presented here gives a comparative study by using simultaneously harmonic domain and geometric algebra in circuits involving disturbances for both sinusoidal and non-sinusoidal excitations, to prove the authenticity of geometric algebra in power flow calculations.

A Spectral Decomposition Approach to the Robust Conversion of 4D Rotation Matrices to Double Quaternions

Thomas Federico

Spain

The problem of approximating dual quaternions by double quaternions emerges when trying to approximate 3D displacements by 4D rotations to simplify some problems arising in Robotics and Computer Graphics. This has triggered a renewed interest in 4D rotations. While 3D rotations can be represented using ordinary quaternions, 4D rotations require the use of double quaternions. Analogously to the 3D case, the mapping from double quaternions to rotation matrices cannot be smoothly inverted because it is a 2-to-1 mapping. Moreover, in practical applications where the elements of rotation matrices are noisy, accuracy problems arise. This paper focuses on the different ways of computing the inversion of the mentioned mapping, including the important case in which the rotation matrices are contaminated by uncorrelated noise, and presents a new spectral decomposition approach which compares favorably with all previously presented methods.

From shape operators of embedded manifolds to shape gauges in the theories of fundamental forces

Vedl Šimon

Czechia

In this talk we first recall the description of embedded manifolds by means of the shape operator - a bivector-valued one-form, which realizes the parallel transport by rotating (multi)vectors in the ambient Euclidean space, and which provides the curvature via a commutator product. Inspired by the embedded geometry, in a general vector bundle setup we introduce the concept of a shape gauge. We showcase this approach on the simplest gauge theory - the electromagnetism - and present shape gauges for some usual examples of electromagnetic fields. Finally, we concern ourselves with the relationship of connections on the vector bundle and connections on the tangent bundle of the base manifold. Towards the end we explore the Einstein equations of general relativity stated in terms of the shape gauge variables.

Shape gauges and rotating blades - new variables for fundamental interactions?

Zatloukal Václav

Czechia

Nature's fundamental forces can be mathematically described in terms of connections (and corresponding curvatures) on vector bundles over the spacetime. In this talk we discuss a procedure, which adds extra dimensions to a given vector bundle, thus allowing for a wider class of gauge transformations. Among the possible gauge choices we focus on what we refer to as the "shape gauge". It enjoys the property that the connection coefficients can be expressed as derivatives of a certain multivector-valued potential - the "rotating blade" - and, as a result, the curvature turns out to be given by a simple algebraic function of the shape-gauge connection. This scheme is motivated by (and analogous to) the treatment of embedded manifolds using the shape operator.